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Pseudo-Dirac Solar Neutrinos

Yosef Nir

School of Natural Science, Institute for Advanced Study
Princeton, NJ 08540, USA[1]
nir@ias.edu

Department of Particle Physics
Weizmann Institute of Science, Rehovot 76100, Israel
ftnir@wicc.weizmann.ac.il

Three of the four viable solutions of the solar neutrino problem are consistent with close to maximal leptonic mixing: $\sin^2\theta_{12} = \frac{1}{2}(1-\epsilon)$ with $|\epsilon| \ll 1$. Theoretical models can naturally explain such a situation if approximate horizontal symmetries force a pseudo-Dirac structure on the neutrino mass matrix. An experimental determination of $|\epsilon|$ and $\text{sign}(\epsilon)$ will constrain the structure of the neutrino mass matrix and consequently provide stringent tests of such flavor models. If both $|\epsilon|$ and Δm^2_{12} are known, it will be possible to estimate the mass scale of the pseudo-Dirac neutrinos. Various subtleties related to the diagonalization of the charged lepton mass matrix, to mixing with the third generation, to radiative corrections, and to the kinetic terms are clarified.

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1. Introduction

Three of the four solutions of the solar neutrino (SN) problem require a large mixing angle. For example, the large mixing angle (LMA) MSW solution requires that the neutrino parameters are within the following ranges (at 99% CL) [1-4]:

$$\Delta m_{12}^2 \approx (1.5 - 20) \times 10^{-5}\ eV^2, \qquad \sin^2 2\theta_{12} \approx 0.55 - 1. \tag{1.1}$$

The case of maximal mixing,

$$\sin^2 2\theta_{12} = 1, \tag{1.2}$$

is particularly interesting from the theoretical point of view. It follows from a simple structure of the relevant 2×2 block in the neutrino mass matrix in the basis where the charged lepton mass matrix is diagonal:

$$M_\nu^{(2)} = m \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{1.3}$$

Such a structure is easily obtained in models of horizontal symmetries [5-9] that try to explain the observed smallness and hierarchy in the charged fermion parameters (mass ratios and mixing angles). For example, if the lepton doublets of the first two generations carry an oppsite charge under an Abelian symmetry (and the relevant scalar field is neutral), then $M_\nu^{(2)}$ has the structure (1.3) in the symmetry limit.

Any horizontal symmetry must be broken in Nature. An unbroken horizontal symmetry leads to either degeneracy between fermions of different generations or vanishing mixing angles (see *e.g.* [10] and references therein). The horizontal symmetry still has observable consequences if the breaking parameters are small. Then the low energy effective theory is subject to selection rules that are manifested in the smallness and hierarchy of the flavor parameters. In the case of close-to-maximal mixing, the small breaking leads to a small splitting between the masses of the two neutrinos and to a small deviation from maximal mixing, that is, the two Majorana neutrinos form a *pseudo-Dirac* neutrino:

$$\frac{\Delta m_{12}^2}{m^2} \ll 1, \qquad 1 - \sin^2 2\theta_{12} \ll 1. \tag{1.4}$$

A measurement of these small effects will provide further information about the pattern of symmetry breaking and guide us in the process of selecting among the many presently viable models of horizontal symmetries. (For interesting studies of the implications of solar neutrino measurements for small entries in the neutrino mass matrix, see refs. [11,12].)

Solar neutrino experiments (and, more generally, any oscillation experiments) are sensitive to the mass-squared difference Δm_{12}^2 but not to the masses themselves. On the other hand, they can be sensitive to small deviations from maximal mixing [13-15]. Moreover, matter oscillations (but not vacuum oscillations) are affected differently by $\theta_{12} > \pi/4$ and by $\theta_{12} < \pi/4$, that is, they are sensitive not only to $\sin^2 2\theta_{12}$ but also to $\sin^2\theta_{12}$. In other words, if the solar neutrino problem is solved by one of the large angle solutions, then experiments may provide us with a measurement of the sign and the size of the small parameter ϵ defined by

$$\sin^2\theta_{12} \;=\; \frac{1}{2}(1-\epsilon)\,, \quad (|\epsilon| \ll 1). \tag{1.5}$$

The purpose of this work is to understand the potential lessons for model building in the framework of horizontal symmetries from solar neutrino measurements of ϵ.

The experimental constraints on ϵ have direct implications for the parameters of $M_\nu^{(2)}$, where $M_\nu^{(2)}$ is the neutrino mass matrix in the interaction basis that is defined as follows:

(a) The charged lepton mass matrix is diagonal;

(b) Rotations that involve the third neutrino have been applied to bring the 3×3 mass matrix to a block-diagonal form;

(c) The energy scale is low (it is the scale that is relevant to solar neutrino experiments).

In section 2 we present the constraints on the neutrino parameters when all the conditions (a)-(c) are fulfilled. However, the predictions of approximate horizontal symmetries apply to the basis where the horizontal charges are well defined. In this basis, any of the following might be the case:

(a) Neither the neutrino nor the charged lepton mass matrix is diagonal;

(b) Entries that mix all three generations do not vanish;

(c) The energy scale is high (it is the scale where the horizontal symmetry is spontaneously broken).

We discuss each of the ingredients (a), (b) and (c) in sections 3, 4 and 5, respectively. In section 6 we demonstrate how potentially powerful the constraints on model building are by applying our results to two classes of models: models with an approximate $L_e - L_\mu - L_\tau$ symmetry and models with a horizontal $U(1) \times U(1)$ symmetry where holomorphic zeros are responsible for the neutrino mass hierarchy. Another subtlety relates to the fact that when the heavy fields with masses at or above the horizontal symmetry breaking scale are integrated out, the kinetic terms for the light neutrino fields may deviate from their canonical form. We discuss this issue in section 7. We summarize our results in section 8.

2. The Effective Two Generation Framework

In this section, we express the mixing angle in terms of the mass parameters in the basis where the charged lepton mass matrix is diagonal. This analysis is useful because it is simplest to interpret the experimental results in this basis. We start however with more general considerations that are useful for the following sections as well.

Let us consider a two generation case. The MNS mixing matrix for leptons [16], V, appears in the charged current interactions,

$$-\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \left(\overline{e_L} \quad \overline{\mu_L} \right) \gamma^\mu V \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} W_\mu^- + \text{h.c.}. \tag{2.1}$$

We parametrize it by

$$V = \begin{pmatrix} c & s e^{i\beta} \\ -s & c e^{i\beta} \end{pmatrix}, \tag{2.2}$$

where $c \equiv \cos\theta_{12}$, $s \equiv \sin\theta_{12}$ and the phase β is physical but does not play a role in oscillation experiments.

Given the charged lepton mass matrix M_ℓ and the neutrino mass matrix M_ν in some interaction basis,

$$-\mathcal{L}_M = \left(\overline{e_L} \quad \overline{\mu_L} \right) M_\ell \begin{pmatrix} e_R \\ \mu_R \end{pmatrix} + \left(\nu_e^T \quad \nu_\mu^T \right) M_\nu \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} + \text{h.c.}, \tag{2.3}$$

V can be found from the diagonalizing matrices V_ℓ and V_ν:

$$V = P_\ell V_\ell V_\nu^\dagger, \tag{2.4}$$

where P_ℓ is a diagonal phase matrix. The unitary matrices $V_{\ell L}$ and V_ν are found from

$$V_{\ell L} M_\ell M_\ell^\dagger V_{\ell L}^\dagger = \text{diag}(m_e^2, m_\mu^2), \quad V_\nu M_\nu^\dagger M_\nu V_\nu^\dagger = \text{diag}(m_1^2, m_2^2). \tag{2.5}$$

Our convention for neutrino masses is as follows:

$$\Delta m^2_{12} \; \equiv \; m_2^2 - m_1^2 \; > \; 0. \tag{2.6}$$

In this section we work, without loss of generality, in the basis where the charged lepton mass matrix is diagonal. In this case we have $V = V_\nu^\dagger$. Our interest lies in the case that the neutrino mass matrix in this basis is of the form

$$M_\nu = m \begin{pmatrix} \delta_e & 1 \\ 1 & \delta_\mu \end{pmatrix}, \tag{2.7}$$

where

$$|\delta_e| \ll 1, \qquad |\delta_\mu| \ll 1. \tag{2.8}$$

It is a straightforward calculation to find the mixing angle $\sin\theta_{12}$ in terms of the parameters δ_e and δ_μ:

$$\sin^2\theta_{12} = \frac{1}{2}\left(1-\epsilon\right), \quad \epsilon = \frac{|\delta_\mu|^2 - |\delta_e|^2}{2|\delta_e^* + \delta_\mu|}. \tag{2.9}$$

For the mass-squared difference, we find

$$\frac{\Delta m^2_{12}}{|m|^2} = 2|\delta_e^* + \delta_\mu|. \tag{2.10}$$

Various bounds from solar neutrino measurements on close to maximal mixing can be interpreted as constraints on the size of the small parameters δ_e and δ_μ.

1. A particularly powerful constraint would follow if the experimental data exclude precisly maximal mixing and provide a lower bound on the deviation of $\sin^2 2\theta_{12}$ from unity,

$$1 - \sin^2 2\theta_{12} = \epsilon^2 = \frac{1}{4}\frac{(|\delta_\mu|^2 - |\delta_e|^2)^2}{|\delta_e^* + \delta_\mu|^2}. \tag{2.11}$$

Note that the deviation is quadratic in the small parameters δ_μ and δ_e. Consequently, it is difficult to accommodate large deviations from $\sin^2 2\theta_{12} = 1$. For example, in most models that use approximate horizontal symmetries to explain the structure of the flavor parameters in both the quark and the lepton sectors, the small breaking parameters λ is of the order of (or smaller than) the Cabibbo angle, $\lambda \sim 0.2$. In such models,

$$\delta_e \sim \lambda^p, \quad \delta_\mu \sim \lambda^q, \tag{2.12}$$

where p and q are positive integers. (In supersymmetric models, either of δ_e and δ_μ may vanish.) We can estimate then the deviation from maximal mixing:

$$\epsilon^2 = \begin{cases} \mathcal{O}(\lambda^2) \sim 0.04 & \min(p,q)=1, \\ \mathcal{O}(\lambda^4) \sim 0.001 & \min(p,q)=2, \end{cases} \tag{2.13}$$

and smaller than $\mathcal{O}(\lambda^6) \sim 10^{-4}$ when $\min(p,q) \geq 3$. A lower bound on ϵ^2 of order 10^{-2} (10^{-1}) would strongly suggest that at least one of p and q is ≤ 2 ($= 1$).

2. Another interesting constraint would follow if experiments allow, for example, only $\theta < \pi/4$, that is $\sin^2\theta_{12} < 1/2$. In other words, the experimental data may tell us that the lighter of the two neutrino mass eigenstates should have a larger component of ν_e and the heavier one should have a larger component of ν_μ. (In the opposite situation, there can be no MSW resonance.) As can be seen from eq. (2.9), the following constraint on δ_μ and δ_e would follow:

$$\theta_{12} < \pi/4 \quad \Longrightarrow \quad |\delta_\mu|^2 > |\delta_e|^2. \tag{2.14}$$

In the generic framework of horizontal symmetries described above, (2.14) would exclude all models where $1 \leq p < q$.

3. There is another interesting issue related to pseudo-Dirac neutrinos where a lower bound on the deviation from maximal mixing can play a role. For pseudo-Dirac neutrinos, the mass-squared difference, Δm^2_{12}, is much smaller than the masses-squared themselves, m^2. Since oscillation experiments tell us Δm^2_{12} but not the masses $m_{1,2} \approx |m|$, one may wonder whether the masses could be large and, in particular, large enough to play a role in galaxy formation ($m = \mathcal{O}$(a few eV)) or to be the heavier neutrino playing a role in atmospheric neutrino (AN) oscillations ($m = \mathcal{O}(0.1$ eV$)$).

If experiments find close-to-maximal mixing, and if we interpret this result as implying a pseudo-Dirac structure, then we can deduce relevant information from the *mixing*. Take, for example, the consequences of the following *hypothetical* experimental constraints:

$$\Delta m^2_{12} \ \simeq \ 2 \times 10^{-5} \ eV^2, \quad \sin^2 2\theta_{12} \ \leq \ 0.99. \tag{2.15}$$

(The day-night asymmetry is very sensitive to Δm^2_{12} within the LMA MSW solution [1].) The upper bound on $\sin^2 2\theta_{12}$ leads, via (2.11), to

$$\frac{||\delta_\mu|^2 - |\delta_e|^2|}{|\delta_e^* + \delta_\mu|} \geq 0.2. \tag{2.16}$$

Assuming that there is no fine tuned relation between δ_μ and δ_e (as is the case in Abelian flavor models), we conclude that $\max(|\delta_\mu|,|\delta_e|) \geq \mathcal{O}(0.2)$. Then the measurement of Δm^2_{12} leads, via (2.10), to

$$m \ \leq \ \mathcal{O}(7\times 10^{-3}\ eV). \tag{2.17}$$

One would conclude that the relevant neutrinos play no role in structure formation and that, to solve the atmospheric neutrino problem, we must have $m_3 > m$. Note that such conclusions could not be made on the basis of the Δm^2_{12} constraint alone.

To summarize, in an effective two generation model, the close-to-maximal mixing between pseudo-Dirac neutrinos and their small mass splitting depend on two small parameters in the neutrino mass matrix. If solar neutrino experiments show that the mixing is indeed close-to (but not preciesly) maximal, then the sizes of these parameters would be constrained. Such constraints will provide stringent tests of models of approximate horizontal symmetries. In this framework, they will allow an estimate of the neutrino masses.

3. The Charged Lepton Mass Matrix

In this section, we calculate the mixing angle in terms of the mass parameters in a generic interaction basis where neither of the mass matrices, M_ℓ and M_ν, is diagonal. This is useful for presenting the predictions of theoretical models that apply in the basis where the horizontal symmetry transformation laws are well defined.

The mixing matrix depends on the product $V_\ell V_\nu^\dagger$ (see eq. (2.4)). We parametrize V_ℓ and V_ν in the following way:

$$V_\ell = \begin{pmatrix} c_\ell & s_\ell e^{i\beta_\ell} \\ -s_\ell & c_\ell e^{i\beta_\ell} \end{pmatrix}, \qquad V_\nu = \begin{pmatrix} c_\nu & s_\nu e^{i\beta_\nu} \\ -s_\nu & c_\nu e^{i\beta_\nu} \end{pmatrix}. \tag{3.1}$$

We can express the size of the mixing angle in terms of the four parameters s_ν, s_ℓ, β_ν and β_ℓ:

$$\sin^2\theta_{12} = c_\ell^2 s_\nu^2 + s_\ell^2 c_\nu^2 - 2\mathcal{R}e(c_\ell s_\ell c_\nu s_\nu e^{i(\beta_\ell-\beta_\nu)}). \tag{3.2}$$

We parametrize the neutrino mass matrix in the way given in eq. (2.7). The most general 2×2 charged lepton mass matrix can be written as

$$M_\ell = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}. \tag{3.3}$$

We are interested in the case that $s_\ell \ll 1$. (If s_ℓ is not parametrically suppressed, then there is in general no reason why the mixing would be close to maximal.) The following combination of parameters in (3.3),

$$\delta_\ell \equiv \frac{m_{11} m_{21}^* + m_{12} m_{22}^*}{|m_{21}|^2 + |m_{22}|^2 - |m_{12}|^2 - |m_{11}|^2}, \tag{3.4}$$

is then constrained to be small:

$$s_\ell \simeq |\delta_\ell| \ll 1. \tag{3.5}$$

To leading order in δ_e, δ_μ and δ_ℓ, we find:

$$\sin^2\theta_{12} = \frac{1}{2}(1-\epsilon), \quad \epsilon = \frac{|\delta_\mu|^2 - |\delta_e|^2}{2|\delta_e^* + \delta_\mu|} + 2\mathcal{R}e\left(\delta_\ell \frac{|\delta_e^* + \delta_\mu|}{\delta_e^* + \delta_\mu}\right). \tag{3.6}$$

Note the δ_ℓ-dependent modification to (2.9). In contrast, the mass difference is of course still given by (2.10).

In the previous section, we presented the implications of a measurement of ϵ for the parameters in the basis where charged lepton masses are diagonal. When discussing the parameters in an interaction basis with a generic charged lepton mass matrix, eq. (3.6) leads to the following points:

1. The deviation of $\sin^2 2\theta_{12}$ from unity is now given by

$$1 - \sin^2 2\theta_{12} = \epsilon^2 = \frac{1}{4}\frac{(|\delta_\mu|^2 - |\delta_e|^2)^2}{|\delta_e^* + \delta_\mu|^2} + 2\mathcal{R}e\left(\delta_\ell \frac{|\delta_\mu|^2 - |\delta_e|^2}{\delta_e^* + \delta_\mu}\right) + 4\left[\mathcal{R}e\left(\delta_\ell \frac{|\delta_e^* + \delta_\mu|}{\delta_e^* + \delta_\mu}\right)\right]^2. \tag{3.7}$$

The deviation from maximal mixing is still quadratic in the small parameters. There is, however, an accidental enhancement that may somewhat modify our estimates:

$$\frac{1 - \sin^2 2\theta_\ell}{1 - \sin^2 2\theta_\nu} \sim 16\ \frac{\delta_\ell^2}{\max(\delta_e^2, \delta_\mu^2)}. \tag{3.8}$$

Note that in models of approximate horizontal symmetries, we expect that the smallness of δ_ℓ is a result of a parametric suppression, similar to (2.12):

$$\delta_\ell \sim \lambda^r, \tag{3.9}$$

where r is a positive integer. (In supersymmetric models it could again happen that $\delta_\ell = 0$. In such models, the analysis of the previous section applies.) Suppose that $r < p, q$, that is, δ_ℓ is the least suppressed among the three small parameters of the mass matrices. Then, for example,

$$\epsilon^2 \sim 4|\delta_\ell^2| = \begin{cases} \mathcal{O}(4\lambda^2) \sim 0.2 & r = 1, \\ \mathcal{O}(4\lambda^4) \sim 0.006 & r = 2. \end{cases} \tag{3.10}$$

A lower bound on ϵ^2 of order 10^{-1} would favor models that give $s_\ell \sim \lambda$.

2. The usefulness of an experimental determination of sign(ϵ) ($\sin^2\theta_{12}$ smaller or larger than 1/2) depends on the relative size of the three small parameters. If $p, q < r$, then $|\delta_\mu|, |\delta_e| \gg |\delta_\ell|$. In such a case, sign($\epsilon$) depends on the relative *size* of $|\delta_\mu|$ and $|\delta_e|$ which is predicted by the models and a useful constraint can be derived (eq. (2.14)). On the other hand, if $r \lesssim p, q$, then sign(ϵ) depends on the relative *phase* between δ_ℓ and $(\delta_e^* + \delta_\mu)$. Since generic models of approximate horizontal symmetries do not predict phases, we cannot derive any useful constraint. Even if all the δ parameters are real, sign(ϵ) depends on the relative *sign* between δ_ℓ and $(\delta_e^* + \delta_\mu)$ which is usually not predicted.

3. While δ_ℓ affects the mixing angle, it does not affect the mass splitting between the neutrinos. Consequently, it could be the case that ϵ is accounted for by δ_ℓ, while $|\delta_e^* + \delta_\mu| \ll |\epsilon|$. In this case, the measurement of ϵ cannot be used to put an upper bound on the mass scale of $m_{1,2}$.

Before concluding this section, let us comment on a particular class of supersymmetric models, where there is no degeneracy among the sleptons and the only mechanism to suppress the supersymmetric contributions to lepton flavor changing decays is *alignment* [17-19], that is small mixing angles in the neutralino-lepton-slepton couplings. In such models, there is a strong constraint on s_ℓ (see *e.g.* [20]):

$$\frac{B(\mu \to e\gamma)}{1.2 \times 10^{-11}} \sim \left(\frac{s_\ell}{2 \times 10^{-3}}\right)^2 \left(\frac{100\ GeV}{m(\tilde{\ell})}\right)^4 < 1, \tag{3.11}$$

where $m(\tilde{\ell})$ is the average slepton mass. In these models it is then particularly difficult to explain a large deviation from maximal mixing. If the dominant source of deviation from maximal mixing is s_ℓ, we have

$$1 - \sin^2 2\theta_{12} \simeq 4s_\ell^2 \lesssim 1.6 \times 10^{-5} \left(\frac{m(\tilde{\ell})}{100\ GeV}\right)^4 . \tag{3.12}$$

To summarize: in this section, we analyzed the implications of a measurement of $\epsilon = 1 - 2\sin^2\theta_{12}$ within models that give order of magnitude predictions for the size of the entries of the lepton mass matrices. We find that if the combination of charged lepton parameters that we call δ_ℓ (see (3.4)) is larger or of the same order of magnitude as the neutrino parameters δ_μ and δ_e (see (2.7)), then (a) it is easier to accommodate more significant deviations from maximal mixing, (b) there is little information in sign(ϵ) and (c) the mass scale of the relevant neutrinos is not bounded. (Similar statements were previously made in ref. [21] in the context of a specific class of textures for the Dirac and Majorana mass matrices in the seesaw model.) In contrast, if $|\delta_\ell| \ll |\delta_e|, |\delta_\mu|$, then our statements of the previous section apply: (a) it is difficult to accommodate $\mathcal{O}(0.1)$ deviations from $\sin^2 2\theta_{12} = 1$, (b) sign(ϵ) constrains the relative size of $|\delta_\mu|$ and $|\delta_e|$, and (c) an upper bound on the order of magnitude of the neutrino masses can be obtained.

4. The Three Generation Framework

There are at least three light neutrinos in Nature. In this work, we assume that the three known active neutrinos are the only light neutrinos. In particular, we do not consider here the possibility of a light sterile neutrino. The MNS mixing matrix for three lepton generations, $V_{\rm MNS}$, is defined in a similar way to V of eq. (2.1). We parametrize it by

$$V_{\rm MNS} = R_{23}(\theta_{23}) R_{13}(\theta_{13}) R_{12}(\theta_{12}), \tag{4.1}$$

where $R_{ij}(\theta_{ij})$ denotes a rotation in the ij plane with an angle θ_{ij} and diagonal phase matrices are left implicit. For simplicity of presentation, we ignore CP violation from here on, that is, we take all our parameters to be real. The extension to the CP violating case is straightforward but cumbersome, and does not change our conclusions. Our convention for neutrino mass eigenstates is given, in addition to (2.6), by

$$|\Delta m^2_{3i}| > \Delta m^2_{21}. \tag{4.2}$$

Note that we allow both positive and negative Δm^2_{3i}: m_3 is not necessarily the heaviest eigenvalue, but instead it is the one most separated from the other two.

The most general structure of a three generation neutrino mass matrix that would lead, in the symmetry limit, to to the effective $M_\nu^{(2)}$ of eq. (1.3) is as follows:

$$M_\nu = mR_{23}(\theta_{23})R_{13}(\theta_{13}) \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & z \end{pmatrix} R_{13}^T(\theta_{13})R_{23}^T(\theta_{23}). \tag{4.3}$$

Clearly, M_ν of eq. (4.3) is diagonalized by $V_{\rm MNS} = R_{23}(\theta_{23})R_{13}(\theta_{13})R_{12}(\pi/4)$ and leads therefore to maximal mixing in the 12 sector.

A combination of CHOOZ [22] and SuperKamiokande results on atmospheric neutrinos [23] implies that $\sin\theta_{13}$ is small [24,25]. In models of approximate horizontal symmetries this phenomenological input is usually accommodated by having $\sin\theta_{13}$ vanish in the symmetry limit. Therefore, in models that explain the data from both solar and atmospheric neutrino measurements, the mass matrices of interest to our investigation have an even simpler form in the symmetry limit [26]:

$$M_\nu = mR_{23}(\theta_{23}) \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & z \end{pmatrix} R_{23}^T(\theta_{23}) = m \begin{pmatrix} 0 & c_{23} & -s_{23} \\ c_{23} & s_{23}^2 z & s_{23}c_{23}z \\ -s_{23} & s_{23}c_{23}z & c_{23}^2 z \end{pmatrix}, \tag{4.4}$$

with $\sin\theta_{23} = \mathcal{O}(1)$.

There is an important point that concerns $\sin\theta_{13}$: even in case that it is small and vanishes in the symmetry limit, it could play an important role in the interpretation of the solar neutrino results. Specifically, if s_{13} is large enough, it may be difficult to set an unambiguous limit on the deviation of s_{12} from maximal mixing. With three neutrino generations and assuming $\Delta m_{12}^2 \ll |\Delta m_{23}^2|$, one has [27]:

$$P_{3\nu}^{\rm MSW} = \cos^4\theta_{13} P_{2\nu}^{\rm MSW} + \sin^4\theta_{13}, \tag{4.5}$$

where $P_{n\nu}^{\rm MSW}$ is the probability of an electron neutrino produced in the sun to emerge from the sun as an electron neutrino, calculated in the n generation framework. For $\sin^2\theta_{13} \ll 1$ and $|\sin^2\theta_{12} - \frac{1}{2}| \ll 1$, eq. (4.5) can be rewritten as follows:

$$P_{3\nu}^{\rm MSW} \simeq \frac{1}{2} - \frac{1}{2}\epsilon - \sin^2\theta_{13} + \mathcal{O}(\sin^4\theta_{13}, \epsilon\sin^2\theta_{13}). \tag{4.6}$$

We learn that a lower bound on $|\epsilon|$ for $\epsilon > 0$ can only be set if

$$\sin^2\theta_{13} \ll \frac{1}{2}\epsilon. \tag{4.7}$$

In the less likely case that $\epsilon < 0$ is preferred, then a lower bound on $|\epsilon|$ can only be strengthened by the presence of a non-vanishing $\sin\theta_{13}$.

It is interesting to note in this context that if the atmospheric flux measurements require $\Delta m^2_{23} > 2\times 10^{-3}\ eV^2$, then the limit on $P(\nu_e \to \nu_e)$ from the CHOOZ experiment [22] requires $\theta_{13} < 13^o$, that is $\sin^2\theta_{13} \leq 0.05$. In such a case, it will be impossible to explain values of $P^{\rm MSW}_{3\nu}$ below 0.45 with $\epsilon = 0$. We learn that a combination of AN constraints on Δm^2_{23} and SN constraints on $P^{\rm MSW}_{3\nu}$ may eventually constrain ϵ for any value of $\sin^2\theta_{13}$ within the reactor bounds. At present the bound on ϵ is valid only for models where $\sin^2\theta_{13}$ is smaller than the bound.

The analysis of this section suggests that, for the purpose of constraining relevant models of horizontal symmetries, it would be useful to present the solar neutrino results on close to maximal mixing as allowed regions in the $\epsilon - \sin^2\theta_{13}$ plane.

5. Radiative Corrections

We consider the effect of radiative corrections on mass matrices that, at a high energy scale Λ, have the form (4.3). In particular, we ask whether at some low energy μ that is relevant to the solar neutrinos, a significant deviation from maximal mixing could be induced by renormalization group evolution (RGE).

In this section, we denote the neutrino mass scale at the high scale Λ, which is assumed to take the form (4.3), by M^0_ν, while the mass matrix at the low scale μ is denoted by M_ν and its form may deviate from (4.3). We also define

$$\hat{M}^0_\nu = m \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & z \end{pmatrix}. \tag{5.1}$$

The important parameter for our purposes is related to the Yuakawa coupling of the tau lepton:

$$\epsilon_\tau \equiv -\frac{g^2_\tau}{(4\pi)^2}(1+\tan^2\beta)\ln\frac{\Lambda}{\mu}. \tag{5.2}$$

Here $g_\tau(1+\tan^2\beta)^{1/2} = m_\tau/v_d$ is the tau Yukawa coupling in the supersymmetric standard model. (Within the SM, one has to replace $(1+\tan^2\beta)$ with $-1/2$.) Define a matrix

$$I_\tau = {\rm diag}(1,1,1+\epsilon_\tau). \tag{5.3}$$

Then, up to universal corrections and negligibly small effects of the muon and electron Yukawa couplings, the renormalized neutrino mass matrix at a scale μ below Λ is given in logarithmic approximation by [28-35,26]

$$M_\nu = I_\tau \cdot M_\nu^0 \cdot I_\tau. \tag{5.4}$$

Obviously, $\hat{M}_\nu^0$ of eq. (5.1) is diagonalized by $R_{12}(\pi/4)$. Equivalently, the 12 rotation that is required to diagonalize M_ν is $R_{12}(\hat{\theta}_{12})$ where $\hat{s}_{12} - \sqrt{2}/2$ vanishes in the limit $\epsilon_\tau \to 0$. The main question that we would like to investigate is whether the difference $\hat{s}_{12} - \sqrt{2}/2$ is of $\mathcal{O}(\epsilon_\tau)$ or $\ll \mathcal{O}(\epsilon_\tau)$. In the first case, radiative corrections could account for rather substantial deviations from maximal mixing, while in the latter they can be safely neglected.

To answer this question, we first define the unitary matrix that diagonalizes M_ν according to

$$M_\nu^{\rm diag} = R_{12}^T(\hat{\theta}_{12}) R_{13}^T(\hat{\theta}_{13}) R_{23}^T(\hat{\theta}_{23}) M_\nu R_{23}(\hat{\theta}_{23}) R_{13}(\hat{\theta}_{13}) R_{12}(\hat{\theta}_{12}). \tag{5.5}$$

It is clear that in the limit $\epsilon_\tau = 0$, we have $\hat{s}_{ij} = s_{ij}$. We define then small parameters δ_{ij} according to

$$\hat{s}_{ij} = s_{ij}(1 + c_{ij}^2 \delta_{ij}). \tag{5.6}$$

We calculate δ_{23} and δ_{13} to $\mathcal{O}(\epsilon_\tau)$ and then go on to find the parametric suppression of δ_{12}. The rotation of M_ν by $R_{13}^T(\hat{\theta}_{13}) R_{23}^T(\hat{\theta}_{23})$ should bring the mass matrix to a block-diagonal form. More explicitly, defining

$$\hat{M}_\nu = R_{13}^T(\hat{\theta}_{13}) R_{23}^T(\hat{\theta}_{23}) M_\nu R_{23}(\hat{\theta}_{23}) R_{13}(\hat{\theta}_{13}), \tag{5.7}$$

we should have $(\hat{M}_\nu)_{13} = (\hat{M}_\nu)_{23} = 0$. We get

$$\begin{aligned} \delta_{23} s_{23} &= \frac{\epsilon_\tau}{1-z^2}[(1+z^2)s_{23} + 2z c_{23} s_{13}], \\ \delta_{13} s_{13} &= \frac{\epsilon_\tau c_{23}}{1-z^2}[(1+z^2)s_{13} c_{23} + 2z s_{23}]. \end{aligned} \tag{5.8}$$

Using these values for δ_{i3}, we find for $\hat{M}_\nu$:

$$\hat{M}_\nu = \begin{pmatrix} -\frac{4z}{1-z^2} c_{23} s_{13}(z s_{23} + c_{23} s_{13})\epsilon_\tau & 1 + (s_{23}^2 + c_{23}^2 s_{13}^2)\epsilon_\tau & 0 \\ 1 + (s_{23}^2 + c_{23}^2 s_{13}^2)\epsilon_\tau & \frac{4}{1-z^2} c_{23} s_{13}(s_{23} + z c_{23} s_{13})\epsilon_\tau & 0 \\ 0 & 0 & z(1 + 2c_{23}^2 c_{13}^2 \epsilon_\tau) \end{pmatrix}. \tag{5.9}$$

We are now in a position to express the 12 mass difference and mixing angle in terms of the parameters of M_ν^0 and ϵ_τ:

$$\begin{aligned}\frac{\Delta m_{12}^2}{m^2} &= 8c_{23}s_{23}s_{13}\epsilon_\tau,\\ \sin\hat\theta_{12} &= \frac{\sqrt{2}}{2}\left[1+\left(\frac{(1+z^2)s_{23}+2zc_{23}s_{13}}{1-z^2}\right)c_{23}s_{13}\epsilon_\tau\right].\end{aligned} \tag{5.10}$$

Our main result is that the RGE-induced deviation from maximal mixing and mass splitting are suppressed by $\epsilon_\tau s_{13}$. Given the experimental constraints on s_{13}, the suppression factor is likely to be very small. In the limit $s_{13}=0$, the leading effects are of order ϵ_τ^2 [26].

We can make even stronger statements if we assume that the contribution to the mass splitting from radiative corrections has no fine-tuned cancellations with other, unrelated contributions, that is, we assume that $\Delta m_{12}^2/m^2 \geq \mathcal{O}(s_{13}\epsilon_\tau)$. (For an early attempt to account for the mass splitting by radiative corrections, see ref. [36].) We can now distinguish between three interesting cases:

1. $z \gg 1$, that is, $\Delta m_{\rm AN}^2 \sim m^2 z^2$. In this case,

$$s_{13}\epsilon_\tau \lesssim \frac{\Delta m_{12}^2}{m^2} \ll \frac{\Delta m_{\rm SN}^2}{\Delta m_{\rm AN}^2}. \tag{5.11}$$

   The radiative corrections drive θ away from $\pi/4$, $\hat s_{12} = \sqrt{2}/2 - c_{23}s_{23}s_{13}\epsilon_\tau$, but by a negligible amount.

2. $z \ll 1$, that is, $\Delta m_{\rm AN}^2 \sim m^2$. In this case,

$$s_{13}\epsilon_\tau \lesssim \frac{\Delta m_{12}^2}{m^2} \sim \frac{\Delta m_{\rm SN}^2}{\Delta m_{\rm AN}^2}. \tag{5.12}$$

   The radiative corrections drive θ away from $\pi/4$, $\hat s_{12} = \sqrt{2}/2 + c_{23}s_{23}s_{13}\epsilon_\tau$, but the effect is smaller than a few percent.

3. $\delta_m \equiv z-1 \ll 1$, that is, $\Delta m_{\rm AN}^2 \sim m^2\delta_m$. In this case, the deviation from maximal mixing is somewhat enhanced by the small value of δ_m:

$$\hat s_{12} = \sqrt{2}/2 - (s_{23}+c_{23}s_{13})c_{23}\frac{s_{13}\epsilon_\tau}{\delta_m}, \tag{5.13}$$

but the effect is still constrained to be small:

$$\frac{s_{13}\epsilon_\tau}{\delta_m} \lesssim \frac{\Delta m^2_{\rm SN}}{\Delta m^2_{\rm AN}}. \tag{5.14}$$

(Note that to naturally induce three quasi-degenerate neutrinos, a non-Abelian horizontal symmetry is required.)

To summarize the results of this section: We find that the contribution from radiative corrections to the deviation from maximal mixing is suppressed beyond the smallness of ϵ_τ. The leading effect is $\mathcal{O}[\epsilon_\tau \times \max(s_{13}, \epsilon_\tau)]$. For three nearly degenerate neutrinos, there is some enhancement and the effect is $\mathcal{O}\left[\frac{\epsilon_\tau}{|z-1|} \times \max(s_{13}, \epsilon_\tau)\right]$. In any case, the same combination of small parameters also contributes to the mass splitting. Consequently, if there is no fine-tuned cancellation, the size of deviation from maximal mixing is constrained to lie below $\Delta m^2_{\rm SN}/\Delta m^2_{\rm AN} \lesssim 10^{-2}$.

6. Explicit Models

To understand the possible implications of close-to-maximal mixing on theoretical model building, we will optimistically assume that in the future the constraint will be strong enough that

$$|(V_\ell)_{12}| \sim \lambda \tag{6.1}$$

will be strongly favored. We examine the consequences of such a constraint on two classes of models in the literature. We find that one class of models will be excluded, while in the other a unique model is singled out that is consistent with all the requirements.

Both classes of models employ an approximate Abelian symmetry. To understand the principles of this framework, let us take the simplest example of a horizontal symmetry, $H = U(1)$, that is broken by a single small parameter. We denote the breaking parameter by λ and assign to it a horizontal charge -1. Then the following selection rules apply:

a. Terms in the superpotential that carry an integer $U(1)$ charge $n \geq 0$ are suppressed by λ^n. Terms with $n < 0$ vanish by holomorphy.

b. Terms in the Kähler potential that carry an integer $U(1)$ charge n are suppressed by $\lambda^{|n|}$.

We are particularly interested in the leptonic Yukawa terms:

$$-\mathcal{L}_Y = Y^\ell_{ij} L_i \bar{\ell}_j \phi_d + \frac{Y^\nu_{ij}}{M} L_i L_j \phi_u \phi_u + \text{h.c.}, \tag{6.2}$$

where $i = 1, 2, 3$ is a generation index, L_i are lepton doublet fields, $\bar{\ell}_j$ are lepton charged singlet fields, and ϕ_u and ϕ_d are the two Higgs fields. The couplings Y_{ij} are dimensionless Yukawa couplings and M is a high energy scale. The Yukawa terms come from the superpotential. If the sum of the horizontal charges in a particular term is a positive integer, then the resulting mass term is suppressed as follows:

$$\begin{aligned} (M_\ell)_{ij} &\sim \langle \phi_d \rangle \lambda^{H(L_i)+H(\bar{\ell}_j)+H(\phi_d)}, \\ (M_\nu)_{ij} &\sim \frac{\langle \phi_u \rangle^2}{M} \lambda^{H(L_i)+H(L_j)+2H(\phi_u)}. \end{aligned} \tag{6.3}$$

Otherwise, *i.e.* if the sum of charges is negative or non-integer, the Yukawa coupling vanishes. We use the $\sim$ sign to emphasize that there is an unknown, independent, order one coefficient for each term (except for the relation $(M_\nu)_{ij} = (M_\nu)_{ji}$).

6.1. $L_e - L_\mu - L_\tau$ symmetry

A particulary interesting mass matrix for neutrinos arises in the frameowork of approximate $L_e - L_\mu - L_\tau$ symmetry [37-41] that is broken by small parameters, ε_+ and ε_- of charges $+2$ and -2, respectively [37]. The neutrino mass matrix has the following form:

$$M_\nu \sim \frac{\langle \phi_u \rangle^2}{M} \begin{pmatrix} \varepsilon_- & 1 & 1 \\ 1 & \varepsilon_+ & \varepsilon_+ \\ 1 & \varepsilon_+ & \varepsilon_+ \end{pmatrix}. \tag{6.4}$$

With a 23 rotation followed by a 13 rotation,

$$\tan \theta^\nu_{23} = -\frac{(M_\nu)_{13}}{(M_\nu)_{12}} = \mathcal{O}(1), \tag{6.5}$$

$$\tan \theta^\nu_{13} = -\frac{c_{23} s_{23} [(M_\nu)_{22} - (M_\nu)_{33}] + (c^2_{23} - s^2_{23})(M_\nu)_{23}}{c_{23}(M_\nu)_{12} - s_{23}(M_\nu)_{13}} = \mathcal{O}(\varepsilon_+), \tag{6.6}$$

one brings M_ν to the form

$$M'_\nu \sim m \begin{pmatrix} \max(\varepsilon_-, \varepsilon_+^3) & 1 & 0 \\ 1 & \varepsilon_+ & 0 \\ 0 & 0 & \varepsilon_+ \end{pmatrix}. \tag{6.7}$$

M'_ν has precisely the form we investigated in previous sections. We can easily find then the parametric suppression of the neutrino masses and diagonalizing angles:

$$m_{1,2} = m\left(1 \pm \mathcal{O}[\max(\varepsilon_+, \varepsilon_-)]\right), \quad m_3 = m\mathcal{O}(\varepsilon_+), \tag{6.8}$$

$$s^\nu_{23} = \mathcal{O}(1), \quad s^\nu_{13} = \mathcal{O}(\varepsilon_+), \quad \sin^2 2\theta^\nu_{12} = 1 - \frac{1}{4}\mathcal{O}[\max(\varepsilon_+^2, \varepsilon_-^2)]. \tag{6.9}$$

(Note that ν_3 is the lightest mass eigenstate.)

To find the mixing angles, we need to consider the charged lepton mass matrix. It has the form [37]:

$$M_\ell \sim \langle \phi_d \rangle \begin{pmatrix} \lambda_e & \lambda_\mu \varepsilon_- & \lambda_\tau \varepsilon_- \\ \lambda_e \varepsilon_+ & \lambda_\mu & \lambda_\tau \\ \lambda_e \varepsilon_+ & \lambda_\mu & \lambda_\tau \end{pmatrix}, \tag{6.10}$$

where the λ_i allow for a generic approximate symmetry that acts on the SU(2)-singlet charged leptons. Such a symmetry, however, does not affect the relevant diagonalizing angles:

$$s^\ell_{23} = \mathcal{O}(1), \quad s^\ell_{13} = \mathcal{O}(\varepsilon_-), \quad s^\ell_{12} = \mathcal{O}(\varepsilon_-). \tag{6.11}$$

Eqs. (6.9) and (6.11) lead to the following estimates of the physical mixing angles:

$$\begin{aligned} s_{23} &= \mathcal{O}(1), \\ s_{13} &= \mathcal{O}[\max(\varepsilon_+, \varepsilon_-)], \\ \sin^2 2\theta_{12} &= 1 - \mathcal{O}[\max(\varepsilon_+^2, \varepsilon_-^2)]. \end{aligned} \tag{6.12}$$

A special feature of this model is that we can estimate the ratio between the SN and AN mass-squared differences:

$$\frac{\Delta m^2_{12}}{\Delta m^2_{23}} \sim \max(\varepsilon_+, \varepsilon_-). \tag{6.13}$$

We learn that, in this model,

$$1 - \sin^2 2\theta_{12} = \mathcal{O}[(\Delta m^2_{\rm SN}/\Delta m^2_{AN})^2]. \tag{6.14}$$

A study of Δm^2 and of the deviation of $\sin^2 2\theta_{12}$ from 1 can then lead to the exclusion of this model in the context of the MSW(LMA) scenario [37]. For example, if $\Delta m^2_{\rm SN}/\Delta m^2_{AN} \leq 10^{-2}$ and $1 - \sin^2 2\theta_{12} \geq 0.1$ are established, then the model is excluded.

6.2. Holomorphic zeros

Within the framework of supersymmetric Abelian horizontal symmetries, it was suggested that holomorphic zeros can induce a large 23 mixing together with large 23 mass hierarchy [42]. The horizontal symmetry is $U(1)_1 \times U(1)_2$ with breaking parameters

$$\lambda_1(-1,0), \quad \lambda_2(0,-1); \quad \lambda_1 \sim \lambda_2 \sim \lambda = 0.2. \tag{6.15}$$

We now impose four requirements on the model:

1. Large 23 mixing, $s_{23} \sim 1$.
2. Large hierarchy, $m_2/m_3 \ll 1$.
3. ν_1 and ν_2 form a pseudo-Dirac neutrino, $0 \neq \Delta m^2_{12} \ll m^2_{1,2}$.
4. Large deviation from maximal mixing, $\sin\theta^\ell_{12} \sim \lambda$. (This is the hypothetical constraint from solar neutrino measurements.)

We find that there is a single set of horizontal charge assignments to the Higgs and lepton doublets that is consistent with all four requirements:

$$\phi_u(0,0), \quad \phi_d(0,0), \quad L_1(1,0), \quad L_2(-1,1), \quad L_3(0,0). \tag{6.16}$$

(The choice is single up to trivial shifts by hypercharge which is an exact symmetry of the model, by a Peccei-Quinn symmetry that is an accidental symmetry of the Yukawa sector, and by lepton number which only changes the overall neutrino mass scale and can be absorbed in the parameter M, and up to trivial exchange of $U(1)_1 \leftrightarrow U(1)_2$.) We find then a unique structure for M_ν:

$$M_\nu \sim \frac{\langle\phi_u\rangle^2}{M}\begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 0 & 0 \\ \lambda & 0 & 1 \end{pmatrix}. \tag{6.17}$$

It leads to

$$\frac{\Delta m^2_{12}}{\Delta m^2_{23}} \sim \lambda^3, \quad \frac{\Delta m^2_{12}}{m^2_{1,2}} \sim \lambda. \tag{6.18}$$

To have large 23 mixing and large enough deviation from maximal 12 mixing, together with acceptable charged lepton mass hierarchy, we can choose, for example,

$$\bar\ell_1(3,4), \quad \bar\ell_2(3,2), \quad \bar\ell_3(3,0). \tag{6.19}$$

We find

$$M_\ell \sim \langle\phi_d\rangle \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^7 & \lambda^5 & \lambda^3 \\ \lambda^7 & \lambda^5 & \lambda^3 \end{pmatrix}. \tag{6.20}$$

We get for the charged lepton masses

$$m_\tau/\langle\phi_d\rangle \sim \lambda^3, \quad m_\mu/m_\tau \sim \lambda^2, \quad m_e/m_\mu \sim \lambda^3, \tag{6.21}$$

and for the MNS mixing angles:

$$\sin^2 2\theta_{12} = 1 - \mathcal{O}(\lambda^2), \quad s_{23} \sim 1, \quad s_{13} \sim \lambda. \tag{6.22}$$

7. Non-Canonical Kinetic Terms

Models with horizontal symmetries predict the structure of the mass matrices in the basis where the horizontal charges are well defined. This preferred interaction basis can, in general, be different from the basis where the kinetic terms are canonically normalized [18,43,44]. In particular, the kinetic terms for the left-handed lepton doublets L_i ($i = 1,2,3$) can be modified to

$$\sum_{i,j} R^L_{ij} L_i^\dagger \gamma^\mu \partial_\mu L_j. \tag{7.1}$$

The deviation of R^L from unit matrix should of course be consistent with the selection rules of the horizontal symmetry. We consider models with a horizontal $U(1)$ symmetry that is broken by a single small parameter λ of charge -1. Then

$$R^L_{ij} \sim \lambda^{|H(L_i)-H(L_j)|}. \tag{7.2}$$

For simplicity, we will work in this section in the two generation framework and assume that the charged lepton mass matrix is diagonal. We will choose a specific case that is useful for our purposes, namely we will take the horizontal charges of the first two neutrino generation to fulfill

$$H(L_1) > 0, \quad H(L_2) < 0, \quad H(L_1) + H(L_2) > 0. \tag{7.3}$$

This charge assignment leads to a pseudo-Dirac structure:

$$M_\nu = m \begin{pmatrix} b\lambda^{H(L_1)-H(L_2)} & 1 \\ 1 & 0 \end{pmatrix}, \tag{7.4}$$

where $m \sim \frac{\langle\phi_u\rangle^2}{M}\lambda^{H(L_1)+H(L_2)}$ and b is an $\mathcal{O}(1)$ coefficient. If we ignore the possibility of non-canonical kinetic terms, then (7.4) would lead to

$$\epsilon = -\frac{|b|}{2}\lambda^{H(L_1)-H(L_2)}, \qquad \frac{\Delta m_{12}^2}{|m|^2} = 2|b|\lambda^{H(L_1)-H(L_2)}. \tag{7.5}$$

Note that this structure is particularly predictive: it gives not only the order of magnitude of ϵ and $\Delta m_{12}^2/m^2$ but also an exact relation between the two quantities, $\Delta m_{12}^2/m^2 = -4\epsilon$, and the sign of ϵ (which is negative). A measurement of the deviation from maximal mixing would lead to determination of the neutrino masses.

Let us now see if and how can the kinetic terms modify the naive predictions in (7.5). We can rescale the L_i fields to set all diagonal elements of R^L to one. Then

$$R^L = \begin{pmatrix} 1 & a\lambda^{H(L_1)-H(L_2)} \\ a^*\lambda^{H(L_1)-H(L_2)} & 1 \end{pmatrix}, \tag{7.6}$$

where a is an $\mathcal{O}(1)$ coefficient. In order to find the true mass matrix, the fields L should be further redefined:

$$L_i = V_{ij}^L L_j' \tag{7.7}$$

where V^L satisfies

$$V^{L\dagger} R^L V^L = \mathrm{diag}(1,1). \tag{7.8}$$

The matrix R^L is hermitian and positive definite. Therefore, eq. (7.8) has a solution. The ambiguity in the solution under multiplication of V^L from the right by a unitary transformation can be fixed by imposing that V^L is hermitian:

$$V^L = \begin{pmatrix} 1 & -\frac{a}{2}\lambda^{H(L_1)-H(L_2)} \\ -\frac{a^*}{2}\lambda^{H(L_1)-H(L_2)} & 1 \end{pmatrix} + \mathcal{O}\left(\lambda^{2[H(L_1)-H(L_2)]}\right). \tag{7.9}$$

The true mass matrix is then

$$M_\nu' = (V^L)^T M_\nu V^L = m \begin{pmatrix} (b-a^*)\lambda^{H(L_1)-H(L_2)} & 1 \\ 1 & -a\lambda^{H(L_1)-H(L_2)} \end{pmatrix}. \tag{7.10}$$

Consequently, we now have:

$$\epsilon = \frac{-|b|^2 + 2\mathcal{R}e(a^*b)}{2|b-2a^*|}\lambda^{H(L_1)-H(L_2)}, \qquad \frac{\Delta m_{12}^2}{|m|^2} = 2|b-2a^*|\lambda^{H(L_1)-H(L_2)}. \tag{7.11}$$

A comparison of the naive calculation, eq. (7.5), with the correct calculation, eq. (7.11), leads to the following conclusions:

1. The size of the deviation from maximal mixing, $|\epsilon|$, has the same order of magnitude as in the naive estimate. The coefficient of order one changes, but it is not predicted by the model in either case.

2. sign(ϵ) depends on the parameters of the kinetic terms. Unlike the naive calculation, which gave an unambiguous prediction for the sign, we now find dependence on the size and the phase of the order one coefficients. We conclude that, in general, in models where the kinetic terms are normalized according to (7.1), sign(ϵ) does not give a useful constraint.

3. The size of the mass splitting, $\Delta m_{12}^2/m^2$, has the same order of magnitude as in the naive estimate. However, the coefficient of order one changes. In particular, the order of magnitude relation $|\epsilon| \sim \Delta m_{12}^2/m^2$ is maintained, but the exact relation is lost. Consequently, one should be able to extract the order of magnitude of the neutrino masses but not determine them exactly.

8. Conclusions

If the solar neutrino problem is solved by a large mixing angle solution, and if the mixing is established to be close to maximal but not precisely maximal, then interesting constraints for theoretical model building would arise. Specifically, experiments may measure the size and the sign of the small parameter ϵ defined by

$$\sin^2\theta_{12} = \frac{1}{2}(1-\epsilon). \tag{8.1}$$

Flavor models can account for a small ϵ by forcing a pseudo-Dirac structure on the neutrino mass matrix through an approximate horizontal symmetry,

$$M_\nu \sim m\begin{pmatrix} \delta_e & 1 \\ 1 & \delta_\mu \end{pmatrix}, \quad |\delta_e|, |\delta_\mu| \ll 1. \tag{8.2}$$

Our main points are the following:

1. The most powerful constraints would arise if δ_e and/or δ_μ are the dominant sources of ϵ. Then the size of $|\epsilon|$ gives the size of the larger between $|\delta_e|$ and $|\delta_\mu|$ while the sign of ϵ determines which of the two is larger. Moreover, the mass scale of the solar neutrinos (and not only their mass-squared splitting) can be estimated.

2. Radiative corrections are unlikely to play a significant role. They are suppressed by the tau Yukawa coupling, by a loop factor, and by $\sin\theta_{13}$. Moreover, the constraints from the ratio $\Delta m^2_{\rm SN}/\Delta m^2_{\rm AN}$ suggest that the radiative corrections contribute negligibly to the deviation from maximal mixing.

3. If the dominant source of ϵ is a small angle in the diagonalizing matrix for the charged lepton mass matrix, s_ℓ, then $|\epsilon|$ constrains the size of s_ℓ but sign(ϵ) is unlikely to test the theoretical models. The order of magnitude relation between $|\epsilon|$ and $\Delta m^2_{12}/m^2_{1,2}$ is lost.

4. If $\sin\theta_{13}$ is not constrained to be small enough by independent measurements, then one must take into account that the effects attributed to $\epsilon > 0$ in the solar neutrino measurements might actually be induced by $\sin^2\theta_{13}$.

5. In models of horizontal symmetries where the kinetic terms are not necessarily canonically normalized, sign(ϵ) depends on the kinetic terms as well and is unlikely to test the models. The order of magnitude of $|\epsilon|$ is not affected.

It remains to be seen whether future developments in solar neutrino experiments would make a convincing case for the intriguing scenario of pseudo-Dirac neutrinos.

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